



Designation: D5850 – 18

# Standard Test Method for (Analytical Procedure) Determining Transmissivity, Storage Coefficient, and Anisotropy Ratio from a Network of Partially Penetrating Wells<sup>1</sup>

This standard is issued under the fixed designation D5850; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.

## 1. Scope\*

1.1 This test method covers an analytical procedure for determining the transmissivity, storage coefficient, and ratio of vertical to horizontal hydraulic conductivity of a confined aquifer using observation well drawdown measurements from a constant-rate pumping test. This test method uses data from a minimum of four partially penetrating, recommended to be positioned observation wells around a partially penetrating control well.

1.2 The analytical procedure is used in conjunction with the field procedure in Test Method [D4050](#).

1.3 *Limitations*—The limitations of the technique for determination of the horizontal and vertical hydraulic conductivity of aquifers are primarily related to the correspondence between the field situation and the simplifying assumption of this test method.

1.4 *Units*—The values stated in inch-pound units are to be regarded as the standard. The SI units given in parentheses are mathematical conversions, which are provided for information purposes only and are not considered standard.

1.5 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice [D6026](#), unless superseded by this standard.

1.6 The procedures used to specify how data are collected/recorded or calculated in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objective; and it is common practice to increase or reduce the significant digits of reported data to be commensurate with these considerations. It is beyond the scope

<sup>1</sup> This test method is under the jurisdiction of ASTM Committee [D18](#) on Soil and Rock and is the direct responsibility of Subcommittee [D18.21](#) on Groundwater and Vadose Zone Investigations.

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of this standard to consider significant digits used in analysis method or engineering design.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.8 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

## 2. Referenced Documents

2.1 *ASTM Standards*:<sup>2</sup>

[D653 Terminology Relating to Soil, Rock, and Contained Fluids](#)

[D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction](#)

[D4050 Test Method for \(Field Procedure\) for Withdrawal and Injection Well Testing for Determining Hydraulic Properties of Aquifer Systems](#)

[D5473/D5473M Test Method for \(Analytical Procedure\) for Analyzing the Effects of Partial Penetration of Control Well and Determining the Horizontal and Vertical Hydraulic Conductivity in a Nonleaky Confined Aquifer](#)

[D6026 Practice for Using Significant Digits in Geotechnical Data](#)

## 3. Terminology

3.1 *Definitions*:

3.1.1 For definitions of common technical terms in this standard, see Terminology [D653](#).

3.2 The following definitions from Terminology [D653](#) are

<sup>2</sup> For referenced ASTM standards, visit the ASTM website, [www.astm.org](http://www.astm.org), or contact ASTM Customer Service at [service@astm.org](mailto:service@astm.org). For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

\*A Summary of Changes section appears at the end of this standard

used in this standard and are presented for the convenience of the user.

3.2.1 *anisotropy*—having different properties in different directions.

3.2.2 *confined aquifer—in hydrogeology*, an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

3.2.3 *control well—in aquifer testing*, well by which the aquifer is stressed, for example, by pumping, injection, or change of head.

3.2.4 *drawdown [L]—in field aquifer tests*, vertical distance the free water elevation is lowered or the pressure head is reduced due to the removal of free water.

3.2.5 *hydraulic conductivity—in field aquifer tests*, the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow.

3.2.6 *monitoring well (observation well)*, *n—in hydrogeology*, a well installed, usually of small diameter, for measuring water levels, collecting water samples, or determining other groundwater characteristics.

3.2.6.1 *Discussion*—The well may be cased or uncased, but if cased the casing should have openings to allow flow of groundwater into or out of the casing, such as a well screen.

3.2.7 *storage coefficient—in aquifers*, the volume of water that an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head. For a confined aquifer, the storage coefficient is equal to the product of the specific storage and aquifer thickness. For an unconfined aquifer, the storage coefficient is approximately equal to the specific yield.

3.2.8 *transmissivity—in aquifers*, the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit width of the aquifer.

3.2.8.1 *Discussion*—It is equal to an integration of the hydraulic conductivities across the saturated part of the aquifer perpendicular to the flow paths.

### 3.3 Symbols and Dimensions:

3.3.1  $A$ — $K_z/K_r$ , anisotropy ratio [ $nd$ ].

3.3.2  $b$ —thickness of aquifer [ $L$ ].

3.3.3  $C_f$ —drawdown correction factor, equal to the ratio of the drawdown for a fully penetrating well network to the drawdown for a partially penetrating well network ( $W(u)/(W(u) + f_s)$ ).

3.3.4  $d$ —distance from top of aquifer to top of screened interval of control well [ $L$ ].

3.3.5  $d'$ —distance from top of aquifer to top of screened interval of observation well [ $L$ ].

3.3.6  $f_s$ —incremental dimensionless drawdown component resulting from partial penetration [ $nd$ ].

3.3.7  $K$ —hydraulic conductivity [ $LT^{-1}$ ].

3.3.7.1 *Discussion*—The use of symbol  $K$  for the term hydraulic conductivity is the predominant usage in groundwa-

ter literature by hydrogeologists, whereas the symbol  $k$  is commonly used for this term in the rock and soil mechanics literature.

3.3.8  $K_o$ —modified Bessel function of the second kind and zero order.

3.3.9  $K_r$ —hydraulic conductivity in the plane of the aquifer, radially from the control well (horizontal hydraulic conductivity) [ $LT^{-1}$ ].

3.3.10  $K_z$ —hydraulic conductivity normal to the plane of the aquifer (vertical hydraulic conductivity) [ $LT^{-1}$ ].

3.3.11  $l$ —distance from top of aquifer to bottom of screened interval of control well [ $L$ ].

3.3.12  $l'$ —distance from top of aquifer to bottom of screened interval of observation well [ $L$ ].

3.3.13  $Q$ —discharge [ $L^3T^{-1}$ ].

3.3.14  $r$ —radial distance from control well [ $L$ ].

3.3.15  $S$ —storage coefficient [ $nd$ ].

3.3.16  $s$ —drawdown observed in partially penetrating well network [ $L$ ].

3.3.17  $s_f$ —drawdown observed in fully penetrating well network [ $L$ ].

3.3.18  $T$ —transmissivity [ $L^2T^{-1}$ ].

3.3.19  $t$ —time since pumping began [ $T$ ].

3.3.20  $u$ — $(r^2S)/(4Tt)$  [ $nd$ ].

3.3.21  $W(u)$ —an exponential integral known in hydrology as the Theis well function of  $u$  [ $nd$ ].

## 4. Summary of Test Method

4.1 This test method makes use of the deviations in drawdown near a partially penetrating control well from those that would occur near a control well fully penetrating the aquifer. In general, drawdown within the screened horizon of a partially penetrating control well tends to be greater than that which would have been observed near a fully penetrating well, whereas the drawdown above or below the screened horizon of the partially penetrating control well tends to be less than the corresponding fully penetrating case. Drawdown deviations due to partial penetration are amplified when the vertical hydraulic conductivity is less than the horizontal hydraulic conductivity. The effects of partial penetration diminish with increasing distance from the pumped well, becoming negligible at a distance of about  $1.5b/(K_z/K_r)^{1/2}$ . This test method relies on obtaining drawdown measurements at a minimum of two locations within this distance of the pumped well and at each location obtaining data from observation wells completed to two different depths. At each location, one observation well should be screened at about the same elevation as the screen in the pumped well, while the other observation well should be screened in sediments not screened by the pumped well.

4.2 According to Theis (1),<sup>3</sup> the drawdown around a fully penetrating control well pumped at a constant rate and tapping a homogeneous, confined aquifer is as follows:

<sup>3</sup> The boldface numbers given in parentheses refer to a list of references at the end of the text.

$$s_f = \frac{Q}{4\pi T} W(u) \quad (1)$$

where:

$$W(u) = \int_u^{\infty} \frac{e^{-x}}{x} dx \quad (2)$$

4.2.1 Drawdown near a partially penetrating control well pumped at a constant rate and tapping a homogeneous, anisotropic, confined aquifer is presented by Hantush (2, 3, 4):

$$s = \frac{Q}{4\pi T} (W(u) + f_s) \quad (3)$$

According to Hantush (2, 3, 4), at late pumping times, when  $t > b^2 S / (2TA)$ ,  $f_s$  can be expressed as follows:

$$f_s = \frac{4b^2}{\pi^2(l-d)(l'-d')} \sum_{n=1}^{\infty} \left( \frac{1}{n^2} \right) K_o \left( \frac{n\pi r \sqrt{K_z/K_r}}{b} \right) \quad (4)$$

$$\left[ \sin \left( \frac{n\pi i}{b} \right) - \sin \left( \frac{n\pi d}{b} \right) \right] \left[ \sin \left( \frac{n\pi l'}{b} \right) - \sin \left( \frac{n\pi d'}{b} \right) \right]$$

4.2.2 For a given observed drawdown, it is practicable to compute a correction factor,  $C_f$ , defined as the ratio of the drawdown for a fully penetrating well to the drawdown for a partially penetrating well:

$$C_f = \frac{W(u)}{W(u) + f_s} \quad (5)$$

The observed drawdown for each observation well may be corrected to the fully penetrating equivalent drawdown by multiplying by the correction factor:

$$s_f = C_f s \quad (6)$$

The drawdown values corresponding to the fully penetrating case may then be analyzed by conventional distance-drawdown methods to compute transmissivity and storage coefficient.

4.2.3 The correction factors are a function of both transmissivity and storage coefficient, that are the parameters being sought. Because of this, the test method relies on an iterative procedure in which an initial estimate of  $T$  and  $S$  are made from which initial correction factors are computed. Using these correction factors, fully penetrating drawdown values are computed and analyzed using distance-drawdown methods to determine revised values for  $T$  and  $S$ . The revised  $T$  and  $S$  values are used to compute revised correction factors,  $C_f$ . This process is repeated until the calculated  $T$  and  $S$  values change only slightly from those obtained in the previous iteration.

4.2.4 The correction factors are also a function of the anisotropy ratio,  $A$ . For this reason, the calculations described above must be performed for several different assumed anisotropy ratios. The assumed anisotropy value that leads to the best solution, that is, best straight line fit or best curve match, is deemed to be the actual anisotropy ratio.

## 5. Significance and Use

5.1 This test method is one of several available for determining vertical anisotropy ratio. Among other available methods are Weeks ((5); see Test Method D5473/D5473M), that relies on distance-drawdown data, and Way and McKee (6), that utilizes time-drawdown data. An important restriction of

the Weeks distance-drawdown method is that the observation wells need to have identical construction (screened intervals) and two or more of the observation wells need to be located at a distance from the pumped well beyond the effects of partial penetration. The procedure described in this test method general distance-drawdown method, in that it works in theory for most observation well configurations incorporating three or more wells, provided some of the wells are within the zone where flow is affected by partial penetration.

### 5.2 Assumptions:

5.2.1 Control well discharges at a constant rate,  $Q$ .

5.2.2 Control well is of infinitesimal diameter and partially penetrates the aquifer.

5.2.3 Data are obtained from a number of partially penetrating observation wells, some screened at elevations similar to that in the pumped well and some screened at different elevations.

5.2.4 The aquifer is confined, homogeneous and areally extensive. The aquifer may be anisotropic, and, if so, the directions of maximum and minimum hydraulic conductivity are horizontal and vertical, respectively.

5.2.5 Discharge from the well is derived exclusively from storage in the aquifer.

5.3 *Calculation Requirements*—Application of this method is computationally intensive. The function,  $f_s$ , shown in (Eq 4) must be evaluated numerous times using arbitrary input parameters. It is not practical to use existing, somewhat limited, tables of values for  $f_s$  and, because this equation is rather formidable, it may not be easily tractable by hand. Because of this, it is assumed the practitioner using this test method will have available a computerized procedure for evaluating the function  $f_s$ . This can be accomplished using commercially available mathematical software including some spreadsheet applications, or by writing programs. (7)

NOTE 1—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

NOTE 2—Most fractured (unconfined) aquifers, even noncarbonates, will have some form of convergent flow to master fissures or channels (Worthington et al., 2016). A relationship is known to occur in carbonates where potentiometric troughs correspond with sub-surface conduits or channels (Quinlan and Ewers, 1989).

## 6. Apparatus

6.1 Apparatus for withdrawal tests is given in Test Method D4050. The apparatus described below are those components of the apparatus that require special attributes for this specific test.

6.2 *Construction of the Control Well*—Screen the control well through only part of the vertical extent of the aquifer to be tested. The exact distances from the top of the aquifer to the top and bottom of the pumped well screen interval must be known.

6.3 *Construction and Placement of Observation Wells*—The procedure will work for arbitrary positioning of observation